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Properties of Buys-Ballot Estimates for Mixed Model in Time Series Decomposition

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ABSTRACT

This paper discusses the statistical properties of Buys-Ballot estimates of linear trend component in descriptive time series analysis and admits mixed model. The emphasis is to characterize the properties of row, column and overall means and variances of the Buys-Ballot table. The method employed in this paper is the Buys-Ballot procedure developed for choice of appropriate model for decomposition, choice of appropriate transformation and assessment/ estimation of linear trend parameters and seasonal component based on means and variances of the Buys-Ballot table. The Minitab 17.0 version statistical software is also employed in this paper. The Buys-Ballot estimates for mixed model, indicates that the column variances ($\hat{\sigma}_j^2$) depends on the column j only through the square of the seasonal effect S_j^2

Keywords: Choice of Model, Time Series Decomposition, Trend-Cycle Component Mixed Model, Buys-Ballot Table.

1. INTRODUCTION

Descriptive time series analysis display periodic behaviour. A periodic time series data has a pattern which repeats every s periods. One of the most common types of periodic behaviour is seasonal variation and use the letter *s* to denote the length of periodicity. By seasonal variation, we mean variation within a stated period, usually a year. The frequency with which time series data are recorded determines the value assigned to s, the length of the periodic interval. A seasonal time series data, of length *n* is conventionally arranged into *m* rows and *s* columns. By arranging a seasonal series into *m* rows and *s* columns; the rows represent the period/year while the columns represent the seasons. This two-dimensional arrangement of a series is called the Buys-Ballot table (see Table 1). For further details of Buys-Ballot table/procedure see Wei, [1] Iwueze and Nwogu, [2-4] Iwueze and Ohakwe, [5] Nwogu, *et al*, [6] Dozie, [7] and Dozie, *et al*, [8]

The Buys - Ballot table for seasonal time series helps in the assessment of the trend cycle component and seasonal indices. The row averages $(\bar{X}_{i.})$ estimate trend, and the differences $(\bar{X}_{.j} - \bar{X}_{..})$ or the ratio $\left(\frac{\bar{X}_{.j}}{\bar{X}_{..}}\right)$ between the column averages $(\bar{X}_{.j})$ and the overall average $(\bar{X}_{..})$ estimate the seasonal indices. Buys-Ballot procedure can be used to determine; i) the model structure (additive or multiplicative or mixed model) ii) the presence or absence of trend parameters and seasonal indices iii) estimation of trend and seasonal indices iv) assessment of trend parameters and seasonal indices v) estimation of error variation without necessarily decomposing the time series data vi) get over the problem of de-trending a time series before computing the estimates of seasonal effect and vii) Buys-Ballot table has an advantage of computing trend easily. However, the estimation procedure does not take into consideration missing observations, cycles and outliers that might be presented in time series data.

Again, the estimation procedure as developed is for time series data that has stable seasonal pattern and does not perform well in the presence seasonal pattern that are not stable over a period of time.

In proposing the test for choice between mixed and multiplicative models in descriptive time series analysis, Nwogu *et al* [6] and Dozie, et al, [8] assumed that (i) the underlying distribution of the variable, X_{ij}, $i = 1, 2, ..., m$, $j = 1, 2, ..., s$, under study is normal, (ii) the observations in each column,(X_{ij}, $i = 1, 2, ..., m$,) are independent and (iii) that the s-columns are independent. They noted that, one of the limitations of proposed test is the violation of some of the assumptions of Chi-square test. Neither the m observations within each group nor the s- groups are independent because the data under study is time series data

The study is to characterize properties of Buys-Ballot estimates for means and variances in time series decomposition, which take into consideration the mixed model structure and linear trend component.

Table 1: Buys - Ballot Table for Seasonal time series

Rows/ Period (i)	Columns (season) j								
	1	2	$\cdots$	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	$\cdots$	X_j	$\cdots$	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	$\cdots$	X_{s+j}	$\cdots$	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	$\cdots$	X_{2s+j}	$\cdots$	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	$\cdots$	$X_{(i-1)s+j}$	$\cdots$	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	$\cdots$	$X_{(m-1)s+j}$	$\cdots$	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	$\cdots$	$T_{.j}$	$\cdots$	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\cdots$	$\bar{X}_{.j}$	$\cdots$	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	$\cdots$	$\hat{\sigma}_{.j}$	$\cdots$	$\hat{\sigma}_{.s}$			$\hat{\sigma}_x$

In this arrangement each time period t is represented in terms of the year/period (i)and column/season (j) Therefore, the row, column and overall totals, averages and variances are defined as

$$T_{i.} = \sum_{j=1}^{s} X_{(i-1)s+j}, \; \bar{X}_{i.} = \frac{T_{i.}}{s}, \qquad \hat{\sigma}_{i.}^2 = \frac{1}{s-1}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{i.}\right)^2$$

$$T_{.j} = \sum_{i=1}^{m} X_{(i-1)s+j}, \qquad \bar{X}_{.j} = \frac{T_{.j}}{m}, \qquad \hat{\sigma}_{.j}^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij} - \bar{X}_{.j}\right)^2 \; T_{..} = \sum_{i=1}^{m}\sum_{j=1}^{s} X_{(i-1)+j}, \qquad \bar{X}_{..} = \frac{T_{..}}{n}, n = ms, \qquad \hat{\sigma}_x^2 = \frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{..}\right)^2$$

2 Row, column and overall means and variances of the Buys-Ballottable for the mixed model when trend-cycle component is linear.

Nwogu, *et al*, [6] and Dozie, *et al*, [8] obtained the row, column and overall means and variances of the Buys-Ballot table for the mixed model given in equations 1-6.

$$\bar{X}_i = \frac{1}{s}\sum_{j=1}^{s} X_{ij} = \left[a + bs(i-1)\right] + \frac{b}{s}\sum_{j=1}^{s} jS_j + \bar{e}_{i.} \quad (1)$$

$$\bar{X}_{.j} = \left[a + b\left(\frac{n-s}{2}\right) + bj\right]S_j + \bar{e}_{.j} \qquad (2)$$

$$\bar{X}_{..} = a + bs\frac{m}{m}\frac{(m-1)}{2} + \frac{b}{s}\sum_{j=1}^{s} jS_j + \bar{e}_{..}$$

$$= a + \frac{b}{2}(n-s) + \frac{b}{s}\sum_{j=1}^{s} jS_j + \bar{e}_{..} \; (3)$$

$$E\left(\hat{\sigma}_i^2\right)=\left[a+bs(i-1)\right]^2 \operatorname{var}\left(S_j\right)+b^2 \operatorname{var}\left(jS_j\right)$$

$$+2b\left[a+bs(i-1)\right]\operatorname{cov}\left(S_j, jS_j\right)+\sigma^2$$

(4)

$$E\left(\sigma_j^2\right)=\frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij}-\bar{X}_{.j}\right)^2$$

$$=b^2\,\frac{n(n+s)}{12}S_j^2+\sigma^2 \qquad (5)$$

$$\sigma_x^2=\frac{n}{n-1}\left\{\frac{b^2(n^2-s^2)}{12}+\left[a^2+ab(n-s)+\frac{b^2(n-s)(2n-s)}{6}\right]Var(S_j)\right.$$

$$\left.+b^2Var\left(jS_j\right)+2b\left[a+b\left(\frac{n-s}{2}\right)\right]Cov(S_j, jS_j)\right\}+\sigma^2$$

(6)

2.1 Properties of Means and Variances of Buys-Ballot Estimates for Mixed Model

1) The periodic meansmimic the shape of the trending curves of the original time series data and contains seasonal component in $C_1=\sum_{j=1}^{s} jS_j$.

2) The seasonal meansalso mimic the shape of the trending curves of the original time series data and contains seasonal component.

3) For periodic variance;i) a functions of trend parameters and seasonal effect ii) a function of row and column specific iii) the expected value involves sum of squares and cross-product trending curves and seasonal indices iv) error variances is not known and needs to be estimated from time series data.

4) For seasonal variance; i) column variances ($\hat{\sigma}_j^2$) depends on the column j only through the square of the seasonal effect S_j^2 ii) a constant multiple of the square of seasonal component iii) a function of slope and seasonal effect iv)a function of column specific v) the error variance is assumed equal to 1 vi) it is the only one (among the variances) that is easily amenable to statistical test.

5) For overall variance; i) a function of weighted average of the square of the seasonal component. ii) expected value of the row variance involves sum of squares and cross-products of trending parameters and seasonal components iii) error variance is not known and requires to be estimated from data series. These properties of Buys-Ballot estimates for means and variances are very important in descriptive time analysis when trend cycle component is linear and admits mixed model could be used to determine; i) the model structure for decomposition of any study data ii) the presence and absence of trend parameters iii) assessment and estimation of seasonal indices.

3. Estimation of Trend Parameters.

The row and overall means are used to estimate the parameters of the trend line. For mixed model

$$a-b\left(s-c_1\right)+\left(bs\right)_i \qquad (7)$$

$$\equiv \alpha+\beta_i \qquad (8)$$

where $\alpha=a-b\left(s-c_1\right),\ \beta=bs$ (9)

$$\therefore \hat{a}=\alpha+\hat{b}\left(s-c_1\right) \qquad (10)$$

$$\hat{b}=\frac{\beta}{s} \qquad (11)$$

Again it reduces to $\hat{a}=X_{..}$ (12)

when there is no trend. That is, when $b=0$, a is estimated using the grand mean $\bar{X}_{..}$

3.1 Estimates of $S_j,\ j=1,2,...,s$

The estimates of the seasonal indices are obtained from Table1 for the mixed model, as

$$\bar{X}_{.j}=\left[a+b\left(\frac{n-s}{2}\right)+b_j\right]S_j \qquad (13)$$

$$\equiv\left[\alpha+\beta_j\right]+S_j \qquad (14)$$

Where $\alpha=a+b\left(\frac{n-s}{2}\right), \qquad \beta=b$ (15)

$$\therefore \hat{S}_j = \frac{\bar{X}_{.j}}{a + b\left(\frac{n-s}{2}\right) + b_j} \quad (16)$$

when there is no trend $b = 0$, we obtain from (16) that

$$\hat{S}_j = \frac{\bar{X}_{.j}}{\bar{X}_{..}} \quad (17)$$

Table 2: Estimates of trend and seasonal Indices

Parameter	Mixed model
a	$\alpha + \hat{b}(s - c_1)$
b	$\frac{\beta}{s}$
S_j	$\frac{\bar{X}_{.j}}{a + b\left(\frac{n-s}{2}\right) + b_j}$

Table 3: Estimates of trend and Seasonal Indices when there no trend (b=0)

Estimate	Mixed model
$\bar{X}_i$	$a + \bar{e}_i$
$\bar{X}_{.j}$	$a + \bar{e}_{.j}$
$\bar{X}_{..}$	$a + \bar{e}_{..}$
S_j	$\frac{\bar{X}_{.j}}{a + \bar{e}_{.j}}$

Nwogu, *et al*, [6] and Dozie, *et al*, [8] proposed chi-square test in seasonal variances of the Buys-Ballot table. The test has shown to be quite efficient and successful in choosing between mixed and multiplicative models for decomposition of any study data. Hence, null hypothesis to be tested is,

H_0: $\sigma_j^2 = \sigma_{0j}^2$

and it is mixed model, against the alternative

H_1: $\sigma_j^2 \neq \sigma_{0j}^2$

and it is not mixed model, where

$\sigma_j^2 = (j = 1, 2, ..., s)$ is the original variance of the jth column.

$$\sigma_{0j}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \quad (18)$$

and σ_1^2 is the error variance, assumed equal to 1.

They noted that the statistic

$$\chi_{jc}^2 = \frac{(m-1)\sigma_j^2}{\sigma_{0j}^2}, \quad j = 1, 2, \ldots s \quad (19)$$

follows the chi-square distribution with $m-1$ degrees of freedom and the sum;

$$\chi_c^2 = \sum_{j=1}^{s} \frac{(m-1)\sigma_j^2}{\sigma_{0j}^2}, \quad (20)$$

They also noted that the interval $\left[\chi^2_{\frac{\alpha}{2}, s(m-1)}, \chi^2_{1-\frac{\alpha}{2}, s(m-1)}\right]$ contains the statistic (20) with 100 (1- α)% degree of confidence

For the purpose of calculation of σ_{0j}^2, both b and S_j are obtainable from column

Mean $\bar{X}_{.j} = \left[a + b\left(\frac{n-s}{2}\right) + bj\right] * S_j$,

rewritten as

$$\equiv [\alpha + \beta j] * S_j \quad (21)$$

where, $\alpha = a + b\left(\frac{n-s}{2}\right)$, $\beta =$ b

Estimates of α and β are obtainable from the regression of $\bar{X}_{.j}$ on j and estimates of $S_{.j}$ is

$$\hat{S}_j = \frac{\bar{X}_j}{\hat{\alpha} + \hat{\beta}_j} \quad (22)$$

3.1 Results from the Mixed Model

The simulated time series employed in this paper consists 100 data sets of 120 observations each simulated from the mixed model: $X_t = (a + bt) \times S_t + e_t$, using the MINITAB 17.0 version statistical software. The trend-cycle component is used with *a=1*, *b=0.02*, $e_t \sim N(0, 1)$ and $S_1=1.15$, $S_2=0.92$, $S_3=0.96$, $S_4=0.78$, $S_5=0.73$, $S_6=1.12$, $S_7=1.14$, $S_8=1.12$, $S_9=1.17$, $S_{10}=1.14$, $S_{11}=0.83$, $S_{12}=0.82$, $S=12.00$. Each time series data of 120 observations has been arranged as monthly data (s = 12) for 10 years (m = 10) in Buys-Ballot table of seasonal time series. The decision rule is to reject mixed model, if the test statistic in (20) lies outside the interval

$\left[\chi^2_{\frac{\alpha}{2},s(m-1)}, \chi^2_{1-\frac{\alpha}{2},s(m-1)}\right]$ or do not rejected it otherwise. At 5% level of significance, the critical values are, for $s(m-1) = 108$ degrees of freedom, equal to 70.1 and 129.6. The calculated values of the test statistic from the simulated time series data are given in Table 4.When compared with the interval70.1 and 129.6, the test statistic lie within the interval in 100 out of the 100 simulations. This shows that the test identified the mixed model successfully in 100% of the times.

Table 4: Calculated Chi-Square for Mixed Model(The critical valuesfor s(m – 1) = 108 degree of freedom are 70.1 and 129.6)

	Series														
S/N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
χ^2_c	117.15	100.12	125.27	98.23	106.93	94.81	100.37	104.13	107.71	118.42	103.92	108.10	112.31	98.76	100.96
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
χ^2_c	94.12	88.70	113.18	100.92	116.37	125.07	121.62	118.91	101.02	118.95	101.39	96.44	105.95	102.41	111.86
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
χ^2_c	105.45	102.30	108.55	107.66	109.24	100.66	97.40	109.61	100.71	107.96	104.37	102.82	101.15	89.89	102.80
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
χ^2_c	102.93	98.54	118.68	120.17	104.56	115.38	102.78	102.31	104.01	97.79	103.87	105.54	110.31	109.30	111.81
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
χ^2_c	104.22	101.90	94.20	118.06	106.37	111.31	104.40	104.74	116.28	113.39	111.96	103.20	109.18	99.11	84.64
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
χ^2_c	117.89	101.73	109.55	96.70	97.91	100.18	103.66	109.54	108.64	107.80	117.89	115.57	118.99	102.56	115.76
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	91	92	93	94	95	96	97	98	99	100					
χ^2_c	114.10	110.49	96.64	98.36	84.64	89.89	89.28	123.05	96.96	115.95					
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept					

4. Concluding Remark

This paper has discussed the properties of Buys-Ballot estimates when trend cycle component is linear in descriptive time series analysis and admits mixed model. The Buys-Ballot table for time series decomposition is adopted for this paper. The study shows that the column variances $(\hat{\sigma}^2_j)$ simply depends on the column j only through the square of the seasonal effect S^2_j.Also, a stimulated example is applied to illustrate the application of the proposed test. Result from the calculated values of the proposed test shows that, the test identified the mixed model successfully in 100 out of the 100 stimulations.

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